

UNCERTAINTY PROPAGATION ANALYSIS OF ARTIFICIAL NEURAL  
NETWORK (ANN) APPROXIMATED FUNCTION USING NUMERICAL AND  
ANALYTICAL METHOD

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I hereby declare that I have checked this project and in my opinion, this project is adequate in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering.

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**STUDENT'S DECLARATION**

I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. The project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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**To my beloved father and mother**

**Mr Mohamad Bin Abdullah**

**Mrs Fatimah Binti Mohamad**

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## ABSTRACT

This thesis is to investigate the uncertainty analysis using numerical sequential perturbation method and analytical Newton approximation method. The objective of this project to propose the a new technique using numerical sequential perturbation in calculating uncertainty propagation compare to the use of analytical Newton approximation method in application where the unknown function is approximated using artificial neural network ANN. The process to determine uncertainty have five step including begin from selected function, randomize the data, function approximation and applied the numerical method in ANN and lastly determine percent of error between numerical with ANN and compare with the analytical method. The ANN was applied in MATLAB software. From the uncertainty analysis, was define that three major figure the end of this project. First figure shown the average error between numerical and analytical method without ANN are **0.03%**. Second figure average error of function approximate the mass flow rate compare the actual value is **0.03%**. The application with numerical method with ANN gives small uncertainty propagation error compare with analytical method where the error is **1.2%** is the last graph of this project. The new technique will be approving to determine the uncertainty analysis using artificial neural network (ANN). This technique also can be applied for application in laboratory or industrial field.

## ABSTRAK

Tesis ini adalah untuk menyiasat analisis ketidakpastian menggunakan kaedah berangka Usikan bersiri dan kaedah analisis pendekatan Newton. Objektif projek ini adalah mencadangkan teknik baru menggunakan kaedah berangka Usikan bersiri dalam menghitung ketidakpastian dibandingkan dengan penggunaan kaedah analisis pendekatan Newton dalam aplikasi di mana fungsi yang tidak diketahui dianggarkan menggunakan rangkaian neural tiruan. Proses untuk menentukan ketidakpastian mempunyai lima langkah, termasuk mulai dari fungsi yang dipilih, merawakkan data, penghampiran fungsi dan menerapkan pendekatan kaedah berangka dalam rangkaian neural tiruan dan terkini menentukan peratus kesalahan antara berangka dengan rangkaian neural tiruan dan bandingkan dengan kaedah analisis. The rangkaian neural tiruan itu diterapkan dalam perisian MATLAB. Dari analisis ketidakpastian, adalah menetapkan bahawa tiga graf utama pada akhir projek ini. Graf pertama menunjukkan purata ralat diantara kaedah berangka dan kaedah analisis tanpa rangkaian neural tiruan adalah 0,03%. Graf Kedua angka purata ralat fungsi laju aliran masa dibandingkan dengan nilai sebenar adalah 0,03%. Aplikasi dengan kaedah berangka dengan rangkaian neural tiruan memberikan nilai ralat ketidakpastian kecil dibandingkan dengan kaedah analisis di mana kesalahan adalah 1,2% adalah graf terakhir daripada projek ini. Teknik baru akan dipersetujui untuk menentukan ketidakpastian analisis menggunakan rangkaian neural tiruan. Teknik ini juga boleh digunakan untuk aplikasi di makmal atau bidang industri.

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**LIST OF SYMBOLS**

%	percent
M	Mass flow
C	Empirical discharge coefficient.
P	Nozzle inlet pressure
A	Nozzle throat area
$\Delta P$	Upstream and Downstream Pressure Different
gc	Gravitational
R	Gas constant for air and humidity
T	Temperature of air at inlet to nozzle

**LIST OF ABBREVIATIONS**

ANN	Artificial Neural Network
SP	Sequential Perturbation
U%	Percent of uncertainty

# CHAPTER 1

## INTRODUCTION

### 1.1 PROJECT BACKGROUND

Uncertainty analysis is a main idea to properly of the result present a systematic approach for identifying, qualifying, and combining the estimates of the error of a measurement in a way that estimates the uncertainty in the final result. Uncertainty analysis can be separated by two case studies which comes specific known function and unknown function. In the first case studies are divided by two, such simple function and complex function. Numerical method is the one of the method to generate the uncertainty analysis. The sequential perturbation technique in the numerical method is easy to implement when the data reduction procedure is automated via a computer program. Analytical Method is other method to find uncertainty analysis. The Newton approximation method technique involves deriving a single formula for the uncertainty in a measurement. The straightforward computation and becomes unwieldy and eventually impractical as the data reduction procedure becomes increasingly complex. (*Theory and design for Mechanical Measurements, Richard S . Figliola*)

Specific unknown function as a measurement data and experiment data will be generate to train it using the matlab software . They can be used to model complex relationships between inputs and outputs or to find patterns in data. The approximation function can be approach by using types of method solution such as numerical method. The Artificial Neural Network will be used and ability to derive meaning from complicated or imprecise data. It can be used to extract patterns and detect trends that are too complex. A neural network learns and does not need to be reprogrammed and ability to learn how to do tasks based on the data given for training or initial experience. In this time no have solution or idea can be solve for the unknown function. This project will be propose to find a new solution about this problem.



## **1.2 PROBLEM STATEMENT**

Generally, uncertainty analysis is to calculate propagation uncertainty estimation for a known function and unknown function. Case 1 study of uncertainty estimation for a known function using two different approaches; analytical approach using Newton Approximation Method and numerical approach using Sequential Perturbation Method. Normally for simple a multivariable function approach by Newton Approximation Method and complex multivariable function approach by Sequential Perturbation Method.

The uncertainty analysis for unknown function is not determined in a specific manner. Case 2 is study of uncertainty estimation for unknown function. In this project, a new approach of uncertainty estimation for unknown function will be proposed. A new method in calculating of uncertainty estimation for an unknown function which is data from experiment or measurement. The proposed method using Numerical Sequential Perturbation Method in calculating uncertainty propagation in application where the unknown function is complex (multivariable) and is approximated using Artificial Neural Network (ANN).

## **1.3 OBJECTIVE OF THE RESEARCH**

The purpose of this project is to show the ability of using Numerical Sequential Perturbation in calculating uncertainty propagation compared to the use of the analytical Newton Approximation Method in application where the unknown function is approximated using Artificial Neural Network (ANN).

## 1.4 SCOPE OF WORK

The approach in the uncertainty analysis of the function will be as follows:

- i. Apply the Artificial Neural Network to function approximation.
- ii. Apply the Matlab software
- iii. Using the Feed-Forward and Back forward network method.
- iv. Used five input and one input from data.

$$\text{v. Use the function } M_{fr} = CA \sqrt{\frac{2g_c p_1 \Delta p}{RT_1}} \quad (1.1)$$

- vi. Compare the result between numerical and analytical method

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 INTRODUCTION

An Artificial Neural Network is a network of simple processors ("units"), each possibly having a (small amount of) local memory. The units are connected by unidirectional communication channel (connections), which carry numeric (as opposed to symbolic) data. The units operate on their local data and on the input they receive via the connection. The design motivation is what distinguishes neural network from other mathematical techniques. A neural network is a processing device, either an **algorithm**, or **actual hardware**, whose design was motivated by the design and functioning of human brains and components thereof.

There are many different types of Neural Networks, each of which has different strengths particular to their applications. The abilities of different networks can be related to their **structure**, **dynamics** and **learning methods**. Neural Networks offer improved performance over conventional technologies in areas which includes: **Machine Vision, Robust Pattern Detection, Signal Filtering, Virtual Reality, Data Segmentation, Data Compression, Data Mining, Text Mining, Artificial Life, Adaptive Control, Optimisation and Scheduling, Complex Mapping** and more.

## 2.2 BRIEF HISTORY OF NEURAL NETWORK

In the early 1940's scientists came up with the hypothesis that neurons fundamental, active cells in all animal nervous systems might be regarded as devices for manipulating binary numbers computers. Early attempts at building ANNs required a great deal of computer power to replicate a few hundred neurons. Consider that an ant's nervous system is composed of over 20,000 neurons and a human being's nervous system consists of over 100 billion neurons.

More recently, ANNs are being applied to an increasing number of complex real world problems, such as pattern recognition and classification, with the ability to generalize and make decisions about imprecise data. They offer solutions to a variety of classification problems such as speech, character, and signal recognition, as well as prediction and system modeling where physical processes are not well understood or are highly complex (Hassoun, 2000).

The study of the human brain dates back thousands of years. But it has only been with the dawn of modern day electronics that man has begun to try and emulate the human brain and its thinking processes. The modern era of neural network research is credited with the work done by neuro-physiologist, Warren McCulloch and young mathematical prodigy Walter Pitts in 1943. McCulloch had spent 20 years of life thinking about the "event" in the nervous system that allowed to us to think, feel, etc. It was only until the two joined forces that they wrote a paper on how neurons might work, and they designed and built a primitive artificial neural network using simple electric circuits. They are credited with the McCulloch-Pitts Theory of Formal Neural Networks. (Haykin, 1994).

The next major development in neural network technology arrived in 1949 with a book, "The Organization of Behavior" written by Donald Hebb. The book supported and further reinforced McCulloch-Pitts's theory about neurons and how they work. A major point brought forward in the book described how neural pathways are strengthened each time they were used. As we shall see, this is true of neural networks, specifically in training a network. (Haykin, 1994).

During the 1950's traditional computing began, and as it did, it left research into neural networks in the dark. However certain individuals continued research into neural networks. In 1954 Marvin Minsky wrote a doctorate thesis, "Theory of Neural-Analog Reinforcement Systems and its Application to the Brain-Model Problem", which was concerned with the research into neural networks. He also published a scientific paper entitled, "Steps Towards Artificial Intelligence" which was one of the first papers to discuss AI in detail. The paper also contained a large section on what nowadays is known as neural networks. In 1956 the Dartmouth Summer Research Project on Artificial Intelligence began researching AI, what was to be the primitive beginnings of neural network research.

Years later, John von Neumann thought of imitating simplistic neuron functions by using telegraph relays or vacuum tubes. This led to the invention of the von Neumann machine. About 15 years after the publication of McCulloch and Pitt's pioneer paper, a new approach to the area of neural network research was introduced. In 1958 Frank Rosenblatt, a neuro-biologist at Cornell University began working on the Perceptron. The perceptron was the first "practical" artificial neural network. It was built using the somewhat primitive and "ancient" hardware of that time. The perceptron is based on research done on a fly's eye. The processing which tells a fly to flee when danger is near is done in the eye. One major downfall of the perceptron was that it had limited capabilities and this was proven by Marvin Minsky and Seymour Papert's book of 1969 entitled, "Perceptrons".(Masters, 1993).

Between 1959 and 1960, Bernard Wildrow and Marcian Hoff of Stanford University, in the USA developed the ADALINE (ADaptive LINear Elements) and MADELINE (Multiple ADaptive LINear Elements) models. These were the first neural networks that could be applied to real problems. The ADALINE model is used as a filter to remove echoes from telephone lines. The capabilities of these model were again proven limited by Minsky and Papert 1969 (Haykin ,1994).

The period between 1969 and 1981 resulted in much attention towards neural networks. The capabilities of artificial neural networks were completely blown out of proportion by writers and producers of books and movies. People believed that such neural networks could do anything, resulting in disappointment when people realized

that this was not so. Asimov's television series on robots highlighted humanity's fears of robot domination as well as the moral and social implications if machines could do mankind's work. Writers of best-selling novels like "Space Odyssey 2001" created fictional sinister computers. These factors contributed to large-scale critique of AI and neural networks, and thus funding for research projects came to a near halt.

An important aspect that did come forward in the 1970's was that of self-organizing maps (SOM's). Self-organizing maps will be discussed later in this project. (Haykin, 1994, pg: 39) In 1982 John Hopfield of Caltech presented a paper to the scientific community in which he stated that the approach to AI should not be to purely imitate the human brain but instead to use its concepts to build machines that could solve dynamic problems. He showed what such networks were capable of and how they would work. It was his articulate, likeable character and his vast knowledge of mathematical analysis that convinced scientists and researchers at the National Academy of Sciences to renew interest into the research of AI and neural networks. At about the same time at a conference in Japan about neural networks, Japan announced that they had again begun exploring the possibilities of neural networks. The United States feared that they would be left behind in terms of research and technology and almost immediately began funding for AI and neural network.

At 1986 saw the first annual Neural Networks for Computing conference that drew more than 1800 delegates. In 1986 Rumelhart, Hinton and Williams reported back on the developments of the back-propagation algorithm. The paper discussed how back-propagation learning had emerged as the most popular learning set for the training of multi-layer perceptrons. With the dawn of the 1990's and the technological era, many advances into the research and development of artificial neural networks are occurring all over the world. Nature itself is living proof that neural networks do in actual fact work. The challenge today lies in finding ways to electronically implement the principals of neural network technology. Electronics companies are working on three types of neuro-chips namely, digital, analog, and optical. With the prospect that these chips may be implemented in neural network design, the future of neural network technology looks very promising.

### **2.2.1 Neurons 101**

The single cell neuron consists of the cell body, or soma, the dendrites, and the axon. The dendrites receive signals from the axons of other neurons. The small space between the axon of one neuron and the dendrite of another is the synapse. The dendrites conduct impulses toward the soma and the axon conducts impulses away from the soma. The function of the neuron is to integrate the input it receives through its synapses on its dendrites and either generate an action potential or not (Chicurel, 1995).

### **2.2.2 ANNs 101**

Neural Networks use a set of processing elements (or nodes) loosely analogous to neurons in the brain (hence the name, neural networks.) These nodes are interconnected in a network that can then identify patterns in data as it is exposed to the data. In a sense, the network learns from experience just as people do. This distinguishes neural networks from traditional computing programs, that simply follow instructions in a fixed sequential order.

Roll your mouse over the picture of the neuron above to see the basic layout or concept behind artificial neural networks. The bottom layer represents the input layer, in this case with 5 inputs. In the middle is something called the hidden layer, with a variable number of nodes. It is the hidden layer that performs much of the work of the network. The output layer in this case has two nodes, representing output values we are trying to determine from the inputs (Hassoun, 2000).

### 2.2.3 Possible Futures of ANNs

The secrets of the human mind still elude us no matter how much we boost processing speed and capacity. That said, neural networks have given us great advancements in tasks such as Optical Character Recognition, financial forecasting and even in medical diagnosis. For any group in which a known interrelationship exists with an unknown outcome there is a possibility that ANNs will be helpful. While the need for computer-based training and e-learning courses grows, the need to develop computer systems that can learn by themselves and improve decision-making will be an ongoing goal of information technology.

## 2.3 NEURAL NETWORK

In general, machine learning involves adaptive mechanism that enable computers to learn from experience, learn by example and learn by analogy. Learning capabilities can improve the performance of an intelligent systems over time. Machine learning mechanism from the basic for adaptive systems. The most popular approaches to machine learning are artificial neural networks and genetic algorithms. This chapter is dedicated to neural networks.

An Artificial Neural Network (ANN) is an information processing paradigm that is inspired by the way biological nervous systems, such as the brain, process information. The key element of this paradigm is the novel structure of the information processing system. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. ANNs, like people, learn by example. An ANN is configured for a specific application, such as pattern recognition or data classification, through a learning process. Learning in biological systems involves adjustments to the synaptic connections that exist between the neurons. This is true of ANNs as well. The complexity of real neurons is highly abstracted when modeling artificial neurons. These basically consist of *inputs* (like synapses), which are multiplied by *weights* (strength of the respective signals), and then computed by a mathematical function which determines the *activation* of the neuron. Another function (which may be the identity) computes the *output* of the artificial